6.4 exponential growth & decay

Suppose you deposit $800 in an account that pays 6.3% annual interest. How much will you have 8 years later if the interest is compounded:

a) annually, 1304.24 b) quarterly, 1319.07 c) monthly 1322.52

d) daily, 1324.21 e) continuously 1324.27

$A(t) = A_0\left(1+\frac{r}{k}\right)^{kt}$

$A_0 =$ initial amount

PV present value

$r =$ interest rate (decimal)

$k =$ # times/yr compounded

$t =$ time (years)

continuously

$A = A_0 e^{rt}$

$e = \lim_{k\to\infty}\left(1+\frac{1}{k}\right)^k$

$\left(1+\frac{1}{10000}\right)^{10000} = 2.718\ldots$

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differential equation for exponential growth or decay

$\frac{dy}{dt} = ky$ solve:

seperate variables:

$\int \frac{1}{y}dy = \int \frac{dy}{y} = \int k\,dt$

$y =$ amount (FV)

$k =$ growth constant

$\frac{dy}{dt} =$ how fast y grows

$e^{\ln|y|} = e^{(kt+c)}$

$|y| = e^{kt+c}$

$y = e^{kt}\cdot e^{c}$

$\boxed{y = c_1 e^{kt}}$

initial conditions

$t=0 \quad y=y_0$

$y_0 = c_1 e^0$

$\boxed{y = y_0 e^{kt}}$

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1. How long will it take to double an investment of 800 dollars compounded continuously at 6%

2. How long to double $18,000?

$$1600 = 800 e^{.06t}$$

$$2 = e^{.06t}$$

$$\ln 2 = \ln e^{.06t}$$

$$11.553 = \frac{\ln 2}{.06} = \cancel{.06}t$$

doubling time T

$$\boxed{T = \frac{\ln 2}{k}}$$

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A bacteria colony begins with 700 bacteria. It doubles every 3.5 hours. How many bacteria will there be after 68 hours

$$y = 700 e^{k \cdot 68}$$ we need k

$$3.5 = \frac{\ln 2}{k}$$ solve for k & substitute

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exp. decay (decrease)

$$\frac{dy}{dt} = -ky$$

solution:

$$y = y_0 e^{-kt}$$

Half-life = H

$$\boxed{H = \frac{\ln 2}{k}}$$

assume $k > 0$

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